

ANALYSIS OF FUNDAMENTAL CONTRADICTIONS OF EFFICIENCY THE CASH FLOW OF PROJECTS

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1. Introduction

It is generally accepted that there is such an estimation rule of an effective project [1–11]:

if $NPV > 0$, $PI > 1$, $IRR > i$ (the project is effective),

if $NPV < 0$, $PI < 1$, $IRR < i$ (the project is not effective),

if $NPV = 0$, $PI = 1$, $IRR = i$ (zero effectiveness),

where NPV is a net present value, PI – stands for profitability index, IRR – means internal rate of return and i is a discount rate.

In the article published in 2014 [1] we demonstrated that discount indices of project effectiveness traditionally used by the majority of theoreticians and analysts did not always represent the actual profitability (unprofitableness) of the project. This article is an attempt to analyze the new similar project variants in theory and practice and offers the way out of such situations.

2. The main material of research

More generally, the economic efficiency of the project can be defined by the following expression [2, p.103]:

$$E = f(t, k, R_1 \dots R_n, A),$$

where E stands for a complex conversion rate; t is a time factor; k means inflation; $R_1 \dots R_n$ means risk factors; A is the project alternativeness.

The function cited above is of little use for practical application because of its multidimensionality. Most of the authors [2; 4–6; 9–10] and analysts use a set of efficiency criteria that describe the feasibility of the project from different angles. They use the index of net present value (NPV) as the main indicator of the project profitability which can be updated taking into account the time factor. The given quantity characterizes the general absolute outcome of the investment activity, its final effect. NPV stands for the difference between the discounted for a moment incomes measures $B(t)$ and expenditures for the realization of the project $C(t)$. In this case t is the number of the year of the project life-cycle. If receipts and expenditures are represented as the intake flow, NPV equals the updated variable of the flow. As the majority of authors point out, the variable NPV is the basis for defining other indicators of efficiency [2; 4–6; 9–10].

If receipts and expenditures are represented as the intake flow, NPV equals the updated variable of the flow. The variable NPV is the basis for defining other indicators of efficiency. In case the intake flow is characterized by the values $R_t = B(t) - C(t)$, which can be both positive and negative, the comparison rate equals i , and we get [3]:

$$NPV = \sum_{t=1}^n \frac{B(t) - C(t)}{(1+i)^t} = \sum_{t=1}^n \frac{R(t)}{(1+i)^t}. \quad (1)$$

When the initial expenses A are singled out during the so-called zero period, Formula 1 is changed in

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the following way:

$$NPV = \sum_{t=1}^n \frac{R(t)}{(1+i)^t} - A. \tag{2}$$

Formulas 1, 2 ,on the one hand, represent the function of the project efficiency, on the other hand, the numerical series of the cash flow calculation. As the effectiveness function, these formulas offer a complex modification of hyperbola or the power function, the form of which depends on the dynamics of the cash flow, while the numerical series is a modification of the geometric progression, the form of which is also dependent on the dynamics of the cash flow. In many ways these conclusions simplify the analysis approach to the project effectiveness in practice.

Let us consider some peculiarities of calculating NPV for the definite kinds of cash flows.

1. If the cash flows of the project are uniformly distributed in time, Rt is a constant = R (constant ordinary annuity). The uniformity of cash flow distribution can be achieved by extending the intervals of planning.

In that case NPV will represent the following numerical series [4]:

$$NPV = -A - R + \left[R + R \frac{1}{1+i} + R \frac{1}{(1+i)^2} + \dots + R \frac{1}{(1+i)^{n-1}} \right] + R \frac{1}{(1+i)^n}. \tag{3}$$

We used square brackets to mark out a classical geometric progression with the general term $q = \frac{1}{1+i} \leq 1$ (series coincide) [4]. After Formula 3 translation we get the following expression:

$$NPV = -A - R + \frac{R}{1-q} - \frac{R}{1-q} q^n + Rq^n = R \frac{(1+i)^n - 1}{i(1+i)^n} - A. \tag{4}$$

If Formula 4 is viewed as the efficiency function where $n \rightarrow \infty$ (perpetual annuity), equation 4 is rearranged in the form:

$$NPV = \frac{R}{i} - A. \tag{5}$$

Let us analyze a similar simple variant. In this case the project efficiency depends on the comparison rate i and combination of R and A . If $A=0$, we have a classical hyperbola (fig.1, the negative values of discount rates are given de benne esse). In this case the stability of the project is absolute, while $IRR \rightarrow \infty$. Can we have such cases in practice?

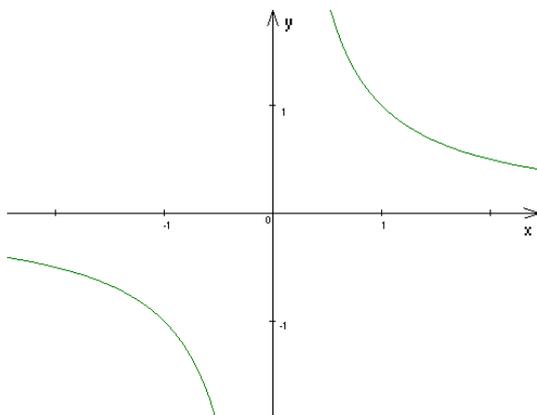


Fig. 1. Classical hyperbole

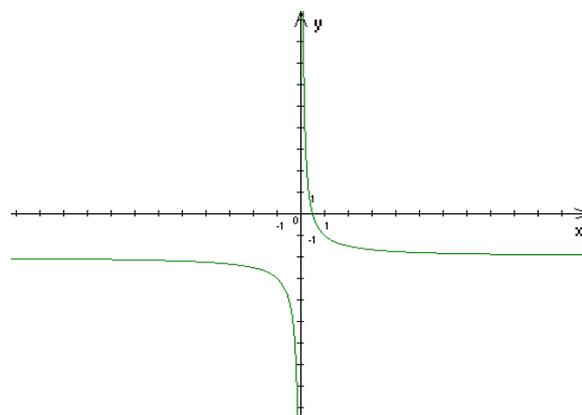


Fig. 2. Hyperbole shift depending on the combination of R and A

Yes, we can, if the initial investments are diffused in the years of life-cycle or are completely lacking (sponsorship, investments out of proceeds of credit, etc).

If the initial investments are used in the zero period the shape of the function of effectiveness depends on the combination of R and A (Fig.2). IRR can be defined with the help of the expression – $NPV = \frac{R}{i} - A = 0 \rightarrow i = \frac{R}{A}$. That is why the forecasting models, based on the cash flow uniformity can have high IRR.

3. Formulation of the problem

Let us analyze the general approach to the effectiveness function using Formula 1. We may have variants in this case as well. The most interesting are: [5]:

- decrease of the cash flow by the end of the life cycle of the project;
- increase of the cash flow by the end of the life cycle of the project;
- fluctuation of the cash flow during the life cycle of the project;
- at last one more variant is possible when $NPV = \sum R(t)$ (net present value is more than total net profit).

Let us study the first case – decrease of the cash flow by the end of the life cycle of the project (Fig. 3).

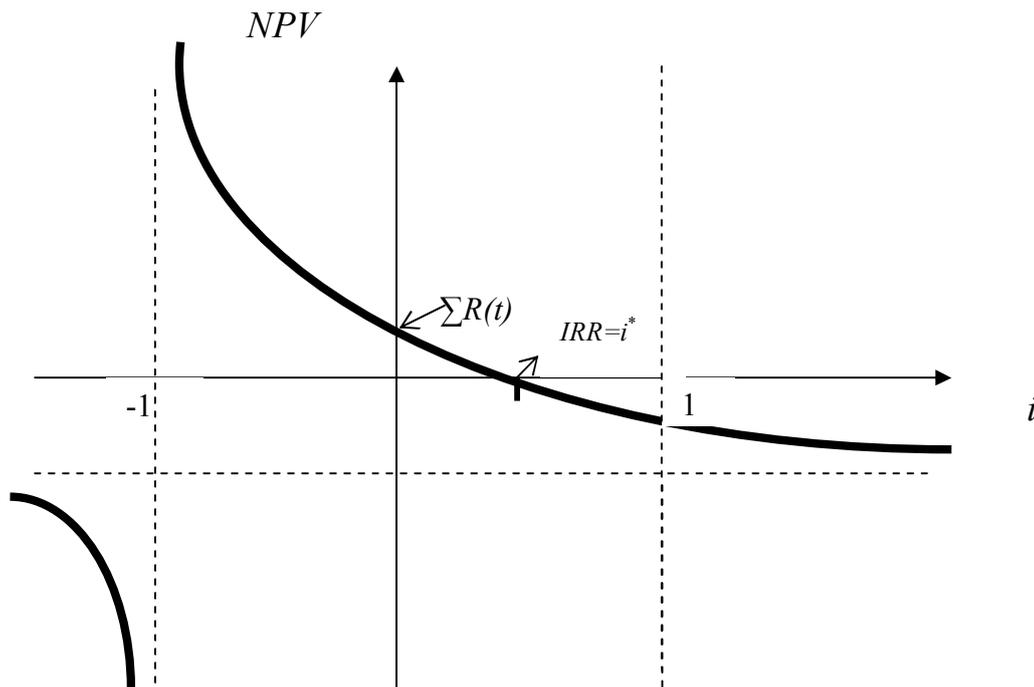


Fig. 3. Reduced cash flow by the end of the project life cycle

The effectiveness curve starts at point $\sum R(t)$ when $i=0$ (we do not take into consideration the negative values of comparison rates in our article) and drops dramatically to the critical value IRR when $NPV=0$. When i goes on rising, $NPV \leq 0$.

The second option is an increase of the cash flow by the end of the life cycle of the project (Fig. 4).

This variant repeats the previous one in its shape but has a much higher point of total cash flow – $\sum R(t)$ and larger IRR (other conditions being equal). Software products for automatic calculations of project efficiency mostly use two models of cash flow growth [1]. In the first model the cash flow growth takes place before the project capacity saturation (Fig. 5) reaching some point (M), then the level of cash flow flattens till the end of the project life cycle. The second model is closely

associated with the life cycle of the project. It means a gradual growth of the cash flows (Fig. 6) to demand saturation (point P), followed by stabilization on this level while maintaining the given value of service, next followed by a decrease to the extent of recession in demand for goods.

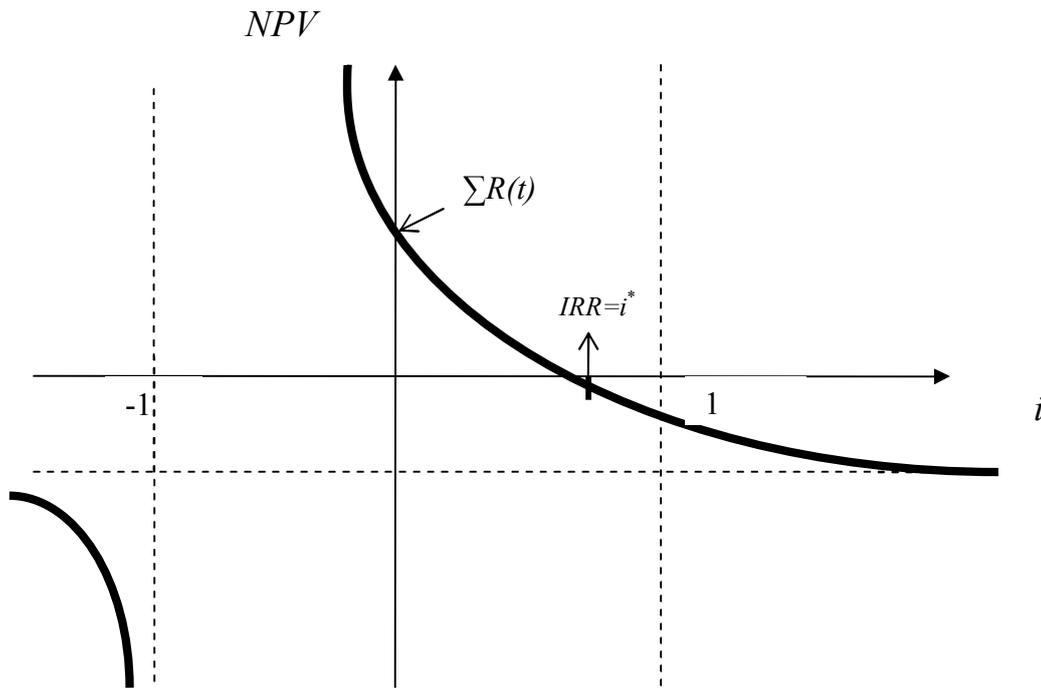


Fig. 4. Increased cash flow by the end of the project life cycle

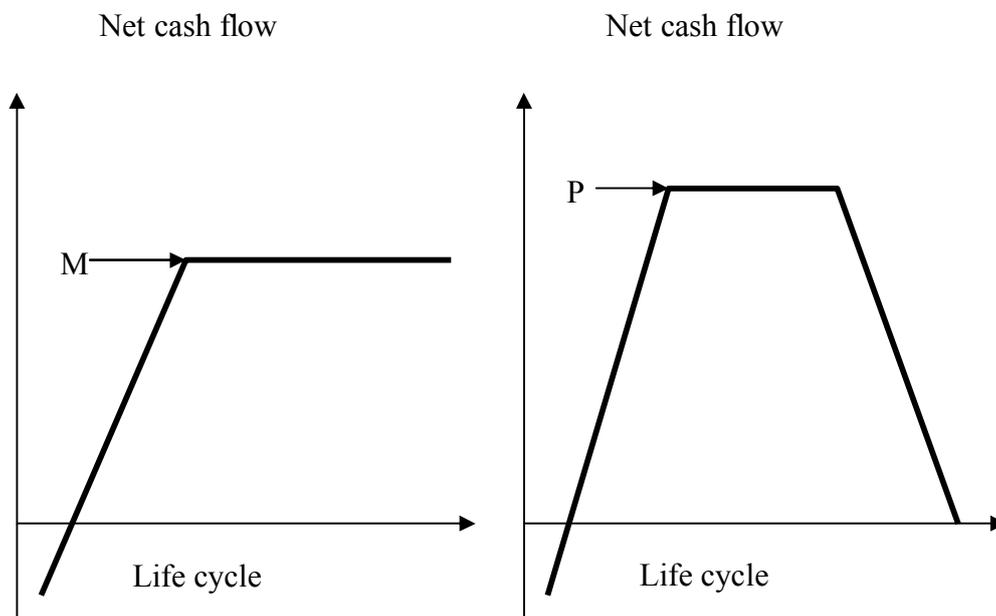


Fig. 5. The saturation of project power Fig. 6. Growth, saturation and falling of cash flows

The first model has an effectiveness curve that is closer to the graphic chart of Fig. 4, the second one is closer to the graphic chart of Fig. 3.

In the third case of the practically investigated cash flows the relation will not be so facile and “correct” as in Fig. 3 and Fig. 4. The picture of the examined relation changes if the members of the flow reverse signs more than once [5], for example, as a result of the fact that some years later after the beginning of the return, modernization of production may be provided, which requires considerable expenses. In this case the graphic chart showing relation between NPV and i will differ

significantly from the graph in Fig. 3 and Fig. 4. Thus, Fig. 7 demonstrates the condition when the variable NPV reverses its sign three times.

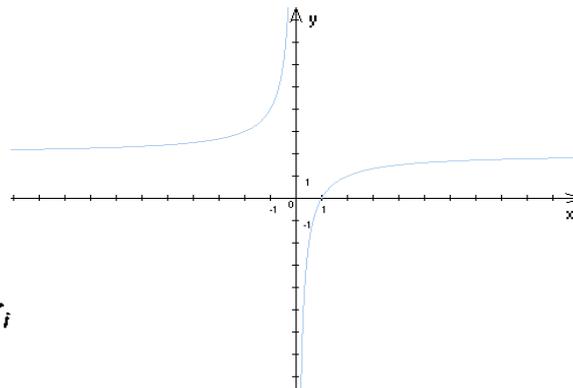
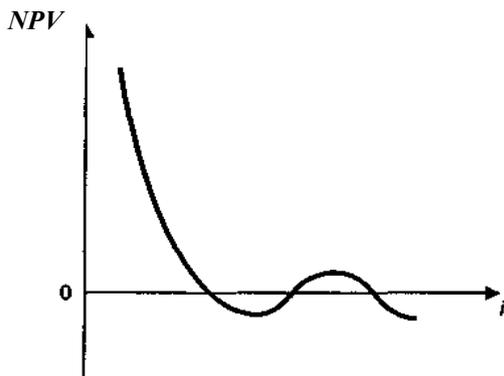


Fig. 7. NPV changes sign three times Fig. 8. Cash flow changes sign from minus to plus

However, in all three cases that we have examined the sign of the cash flow is reversed from the negative to the positive one, in the latter case from minus to plus, then to minus again and so on.

Theoretically the reversed situation is possible when the cash flow reverses a sign from plus to minus (not in the zero period). In such a case we can get the effectiveness curve like the one in Fig. 8.

At the same time there might be situations with calculating NPV, when $NPV \geq \sum R$ (total net cash flow). Such a situation may seem impossible based on expressions 1 and 2. Let us study the situation of illustrative example 1 (Tab. 1).

Tab. 1. Illustrative example of cash flow calculation of project 1

Initial expenses	10	Activities	Periods					Total
			1	2	3	4	5	
Discount rate	0,15	Current expenses	0	0	0	0	50	50
Units of cash flow	Standard unit	Revenue	0	0	40	10	10	60
Life cycle of the project	5							

When the discount rate is on the level of 0,15 (15%), the initial expenses in the zero period are 10 standard units, and the cash flow distribution is as shown in table 1, we have a zero net cash return ($\sum R = -10 - 50 + 40 + 10 + 10 = 0$), however

$$NPV = -10 + \frac{40}{(1+0.15)^3} + \frac{10}{(1+0.15)^4} + \frac{10-50}{(1+0.15)^5} = 2.13.$$

It seems to contradict the fundamental postulate of project efficiency – if $NPV > 0$, the project is effective. However, for crisis project variants the problem situation $\sum R = 0$ remains, which makes the project effective. Let us calculate the effectiveness curve for our example 1 (Tab. 2).

Tab. 2. Calculation of the effectiveness curve of illustrative example 1

<i>i</i>	0.00009	0.001	0.1	0.15	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
<i>NPV</i>	0.0035	0.003	2.04	2.13	1.89	0.93	-0.25	-1.44	-2.5	-3.5	-4.3	-5.0	-5.6

As we see from the table information and the effectiveness curve based on it (Fig. 9), NPV maximizes from 0 at point $i = 0.15$, $NPV = 2.13$, then it drops to point $IRR = 0.38$ and below 0.

However, as our experience has shown, such a situation is quite possible in practice for quite successful projects. If the project has moderate volumes of current expenses, which, for example, are realized against credit or some other ways of borrowing with a considerable delay of credit payment, we may have non-recurring incomes from the project at one of the initial stages. These non-recurrent

incomes can significantly exceed current costs. Credit repayments take place at the end of the project life cycle. In this case it is possible to have the situation $NPV > \sum R > 0$.

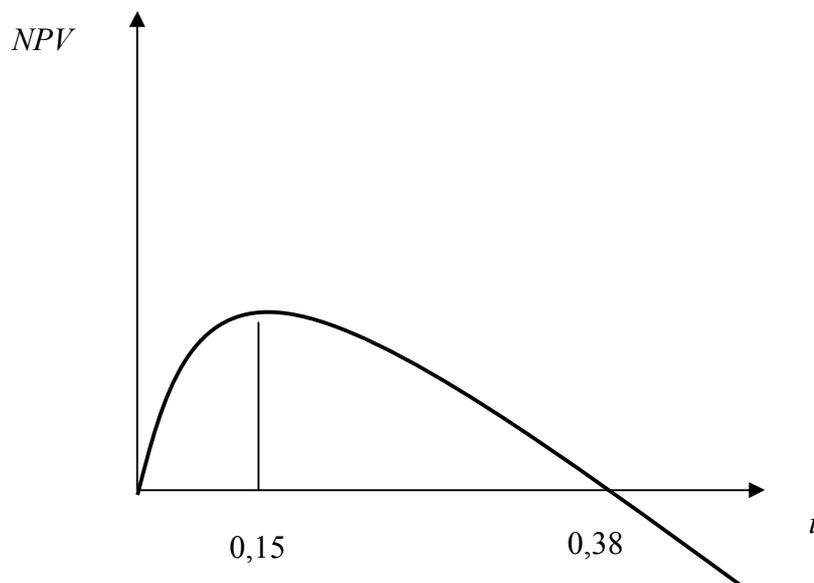


Fig. 9. Efficiency curve of conventional example (Tab. 1)

Let us examine the situation. A credit for capital costs was given for project 2. The credit is at the amount of 30 standard units at annual interest 16.7% for 5 years on condition of paying off at the end of the period. The results of realizing the project with the life cycle of 5 years and the annual comparison rate of 15% are represented in Tab. 3.

Tab. 3. Illustrative example of cash flow calculation of project 2

Initial expenses	30	Activities	Periods					Total
Discount rate	0,15		1	2	3	4	5	
Units of cash flow	Standard units	Current expenses	0	0	0	0	65	65
Life cycle of the project	5	Revenue	50	20	10	0	0	80

According to the credit conditions the amount of payment will be 65 standard units at the end of the fifth year.

In terms of the results of the project we have $\sum R = -30 + 50 + 20 + 10 - 65 = -15$ standard units, and $NPV = -30 + \frac{50}{(1+0.15)^1} + \frac{20}{(1+0.15)^2} + \frac{10}{(1+0.15)^3} - \frac{65}{(1+0.15)^5} = 2.86st.monet.unit$

In this case we have $NPV > 0 > \sum R < 0$. It is a positive NPV in the inefficient project.

Let us calculate the effectiveness curve for our example 2 (Tab. 4).

Tab. 4. Calculation of effectiveness curve of illustrative example 2

<i>i</i>	0,00009	0,001	0,1	0,15	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
<i>NPV</i>	-14,99	-14,79	-0,86	2,86	5,2	7,3	7,47	6,62	5,3	3,79	2,2	0,69	-0,78

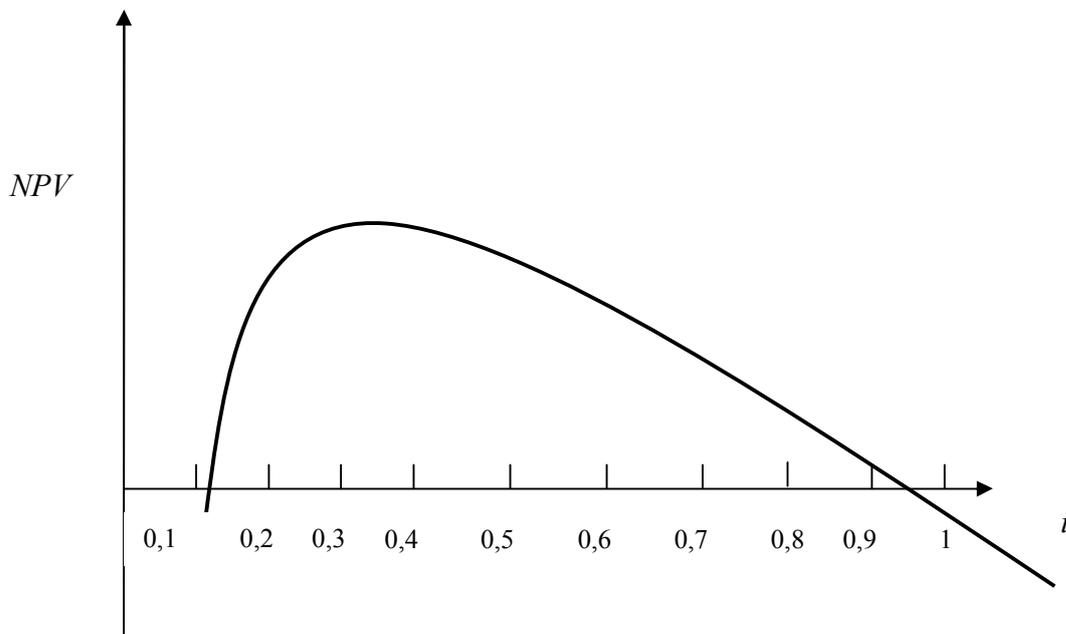


Fig. 10. Efficiency curve of conventional example 2 (Tab. 4)

As we see from the information in the table and the effectiveness curve based on it (Fig. 10), NPV at two points equals 0 (if $i = 0,11$ and $0,946$), the maximum is between the points $i = 0,3$ and $0,4$, $NPV = 7,47$, there may be equality between NPV and $\sum R$ at point $i = 3$.

As the calculations given above show, the measures of project efficiency, based on calculating NPV do not always render the actual effectiveness of the project (variant in Tab. 3).

In our opinion, such a situation can arise as a result of including expenses into the calculation of measures of effectiveness, which discount at the same time with the revenue, thereby they computationally increase the discount level of project profitability (as a result of discounting the negative value of expenses the total rate of return increases. Though, if one accepts the premise of the theory of time value of money [10, p.353], these are the expenses that influence the present value – PV . That is in every period of time t costs $C(t)$ give rise to future earnings $B(t)$. In its semantic loading $C(t)$ is the initial value P for a future earnings-flow during t period. That is why we think it is not advisable to include the costs into the discount part of the measure of project effectiveness.

In order to evaluate the general effectiveness of the project it is possible to offer the index of total discount revenue deducting the total project expenditure during the whole life cycle of the project ($PVNC$):

$$PVNC = \sum_{t=1}^n \frac{R(t)}{(1+i)^n} - \sum_{t=0}^n C(t). \quad (6)$$

Let us analyze the use of formula 5 to evaluate the above mentioned examples of projects.

The zero value of the index $PVNC$ is obtained for the projects in which the total discount revenue equals the total expenditure:

$$\sum_{t=1}^n \frac{R(t)}{(1+i)^n} - \sum_{t=0}^n C(t) = 0 \Rightarrow \sum_{t=1}^n \frac{R(t)}{(1+i)^n} = \sum_{t=0}^n C(t)$$

The calculations showed that for project 1 $\sum R=0$, $NPV= 2.13$, $PVNC = -23.16 < 0$.

For project 2 $\sum R=-15$, $NPV= 2.86$, $PVNC = -29,8 < 0$.

Thus, according to the index $PVNC$, both projects are ineffective because total discount revenues do not cover the total project expenditure.

Tab. 5. Illustrative example of calculating NPV and PVNC of project 3

Initial expenses	80	Activities	Periods					Total
Discount rate	0.15		1	2	3	4	5	
Units of cash flow	Standard units	Current expenses,	0	0	0	0	1000	1000
		General project costs	–	–	–	–	–	1080
Life cycle of the project	5	Revenue	390	350	360	210	240	1550
$\sum R$			470					
NPV			502.7					
PVNC			–0.123					

In practice it is often possible to come across projects which have the growing current cost, connected with its liquidation, at the end of economic life. The numerical illustration of such project 3 is presented in Tab. 5.

As we see from table 5, $NPV=502.7 > \sum R=470$, which completely distorts the fundamental postulates of the theory of time value of money. In fact, the total discount revenues do not cover the total costs of the project $\sum_{t=1}^n \frac{R(t)}{(1+i)^t} = 1079.9 < \sum_{t=0}^n C(t) = 1080$, they are about equal, the index $PVNC = -0,123$ is approximate to zero. Taking into consideration the value $PVNC$, project 3 from the point of view of the theory of time value of money has zero efficiency.

4. Conclusions

1. The rule $NPV > 0$, $PI > 1$, $IRR > i$ is not always true. In some variants of project realization (sponsorship, credit with deferral of payments, other forms of investment at the cost of borrowed funds, as well as projects, which have growing current cost at the end of economic life, the cost exceeds the project revenue, which stops the whole project). This rule does not always represent the real profitability (unprofitableness) of the project. For such projects we suggest counting the index of the total discount revenue with the deduction of total project expenditure during the whole project life cycle – $PVNC$, if its value is below zero. Such a project should be turned down, as total discount revenue does not cover the total project expenditure.
2. The calculation of conversion rate is to be accompanied by the economic analysis of the project cash flow.

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Summary

The article makes an attempt to give a meaning to theoretical and practical calculation problems of cash flow of projects. There are examples given of calculating net discount profit for different in character variants of cash flows of projects.

Keywords: cash flow; comparison rate; effective project; effectiveness curve; net present value.

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